Chapter 6. Simultaneous (Linear) Equations (Including Problems)

Exercise 6(A)

Solution 1:

$8x + 5y = 9 \ldots (1)$
$3x + 2y = 4 \ldots (2)$

$(2) \Rightarrow y = \dfrac{9-8x}{5}$

Putting this value of y in (2)

$3x + 2\left(\dfrac{9-8x}{5}\right) = 4$

$15x + 18 - 16x = 20$
$x = -2$

From (1) $y = \left(\dfrac{9-8x}{5}\right) = \dfrac{9-8(-2)}{5} = \dfrac{25}{5} = 5$

$y = 5$

Solution 2:

$2x - 3y = 7 \ldots (1)$
$5x + y = 9 \ldots (2)$

$(2) \Rightarrow y = 9 - 5x$
Putting this value of y in (1)
$2x - 3(9 - 5x) = 7$
$2x - 27 + 15x = 7$
$17x = 34$
$x = 2$
From (2)
$y = 9 - 5(2)$
$y = -1$

Solution 3:

$2x + 3y = 8 ...(1)$
$2x = 2 + 3y ...(2)$

$(2) \Rightarrow 2x = 2 + 3y$
Putting this value of 2x in (1)
$2 + 3y + 3y = 8$
$6y = 6$
$y = 1$
From (2) $2x = 2 + 3(1)$
$x = \frac{5}{2}$
$x = 2.5$

Solution 4:

The given pair of linear equations are
$0.2x + 0.1y = 25 ....(i)$
$2(x - 2) - 1.6y = 116 ....(ii)$

Consider equation (i)
$0.2x + 0.1y = 25$
$\Rightarrow 0.2x = 25 - 0.1y$
$\Rightarrow x = \frac{(25 - 0.1y)}{0.2} ....(iii)$

Substitute the value of x from equation (iii) in equation (ii).
$2(x - 2) - 1.6y = 116$
$\Rightarrow 2\left(\frac{(25 - 0.1y)}{0.2} - 2\right) - 1.6y = 116$
$\Rightarrow 10(25 - 0.1y) - 4 - 1.6y = 116$
$\Rightarrow 250 - y - 4 - 1.6y = 116$
$\Rightarrow -2.6y = -130$
$\Rightarrow y = 50 ....(iv)$

Substitute the value of y from equation (iv) in equation (iii).
$x = \frac{(25 - 0.1y)}{0.2}$
$\Rightarrow x = \frac{(25 - 0.1(50))}{0.2}$
$\Rightarrow x = \frac{(25 - 5)}{0.2}$
$\Rightarrow x = 100$

$\therefore$ Solution is $x = 100$ and $y = 50$.

Solution 5:

6x = 7y + 7...(1)
7y - x = 8...(2)
(2) $\Rightarrow$ x = 7y - 8
Putting this value of x in (1)
6(7y - 8) = 7y + 7
42y - 48 = 7y + 7
35y = 55

$$y = \frac{11}{7}$$

From (2) $x = 7\left(\frac{11}{7}\right) - 8$

x = 3

Solution 6:

y = 4x -7...(1)
16x- 5y = 25...(2)
(1) $\Rightarrow$ y = 4x - 7
Putting this value of y in (2)
16x - 5 (4x - 7) = 25
16x - 20x + 35 = 25
-4x = -10

$$x = \frac{5}{2}$$

From (1)

$$y = 4\left(\frac{5}{2}\right) - 7$$

$$\Rightarrow y = 10 - 7$$

$$\Rightarrow y = 7$$

y = 10-7=3
Solution is $x = \frac{5}{2}$ and $y = 3$.

Solution 7:

2x + 7y = 39...(1)
3x + 5y = 31...(2)
(1) $\Rightarrow x = \frac{39 - 7y}{2}$
Putting this value of x in (2)

$$3\left(\frac{39 - 7y}{2}\right) + 5y = 31$$

117 - 21y + 10y = 62
-11y = -55
y = 5

From (1) $x = \frac{39 - 7(5)}{2}$

$$x = \frac{4}{2}$$

$$x = 2$$

Solution 8:

The given pair of linear equations are

$1.5x + 0.1y = 6.2$..............(*i*)

$3x - 0.4y = 11.2$..............(*ii*)

Consider equation (*i*)

$1.5x + 0.1y = 6.2$

$\Rightarrow 1.5x = 6.2 - 0.1y$

$\Rightarrow x = \frac{(6.2 - 0.1y)}{1.5}$.........(*iii*)

Substitute the value of x from equation (iii) in equation (ii).

$3x - 0.4y = 11.2$

$\Rightarrow 3\left(\frac{(6.2 - 0.1y)}{1.5}\right) - 0.4y = 11.2$

$\Rightarrow 2(6.2 - 0.1y) - 0.4y = 11.2$

$\Rightarrow 12.4 - 0.2y - 0.4y = 11.2$

$\Rightarrow -0.6y = -1.2$

$\Rightarrow y = 2$........(*iv*)

Substitute the value of y from equation (iv) in equation (iii).

$x = \frac{(6.2 - 0.1y)}{1.5}$

$\Rightarrow x = \frac{(6.2 - 0.1(2))}{1.5}$

$\Rightarrow x = \frac{(6.2 - 0.2)}{1.5}$

$\Rightarrow x = 4$

$\therefore$ Solution is $x = 4$ and $y = 2$.

Solution 9:

Given equations are

$2(x - 3) + 3(y - 5) = 0$ ....(1)

$5(x - 1) + 4(y - 4) = 0$ ....(2)

From (1), we get

$2x - 6 + 3y - 15 = 0$

$\Rightarrow 2x - 21 + 3y = 0$

$\Rightarrow 2x = 21 - 3y$

$\Rightarrow x = \frac{21 - 3y}{2}$

From (2), we get

$5(x - 1) + 4(y - 4) = 0$

$\Rightarrow 5x - 5 + 4y - 16 = 0$

$\Rightarrow 5x + 4y - 21 = 0$ ....(3)

Substituting $x = \frac{21 - 3y}{2}$ in (3), we get

$5\left(\frac{21 - 3y}{2}\right) + 4y - 21 = 0$

$\Rightarrow \frac{105 - 15y}{2} + 4y - 21 = 0$

$\Rightarrow 105 - 15y + 8y - 42 = 0$

$\Rightarrow -7y + 63 = 0$

$\Rightarrow 7y = 63$

$\Rightarrow y = 9$

Substituting $y = 9$ in $x = \frac{21 - 3y}{2}$, we get

$x = \frac{21 - 3(9)}{2} = \frac{21 - 27}{2} = \frac{-6}{2} = -3$

Solution 10:

$\frac{2x+1}{7}+\frac{5y-3}{3}=12$ (given)

$\Rightarrow \frac{3(2x+1)+7(5y-3)}{21}=12$

$\Rightarrow 6x+3+35y-21=252$

$\Rightarrow 6x+35y-18=252$

$\Rightarrow 6x+35y=270$

$\Rightarrow 6x=270-35y$

$\Rightarrow x=\frac{270-35y}{6}$

$\frac{3x+2}{2}-\frac{4y+3}{9}=13$ (given)

$\Rightarrow \frac{9(3x+2)-2(4y+3)}{18}=13$

$\Rightarrow 27x+18-8y-6=234$

$\Rightarrow 27x-8y+12=234$

$\Rightarrow 27x-8y=222$ ....(1)

Substituting $x=\frac{270-35y}{6}$ in (1), we get

$27\left(\frac{270-35y}{6}\right)-8y=222$

$\Rightarrow 7290-945y-48y=1332$

$\Rightarrow -993y=-5958$

$\Rightarrow y=6$

Substituting $y=6$ in $x=\frac{270-35y}{6}$, we get

$x=\frac{270-35\times 6}{6}=\frac{270-210}{6}=\frac{60}{6}=10$

$\therefore$ Solution is $x=10$ and $y=6$

Exercise 6(B)

Solution 1:

$13+2y=9x$...(1)

$3y=7x$...(2)

Multiplying equation no. (1) by 3 and (2) by 2, we get,

$$39+6y=27x \quad \ldots(1)$$

$$6y=14x \quad \ldots(2)$$

$$- \quad - \quad -$$

$$39=13x$$

$$x=3$$

From (2)

$3y=7(3)$

$y=7$

Solution 2:

$3x - y = 23 \ldots(1)$

$$\frac{x}{3} + \frac{y}{4} = 4$$

$4x + 3y = 48 \ldots(2)$

Multiplying equation no. (1) by 3

$$9x - 3y = 69 \quad \ldots(3)$$
$$4x + 3y = 48$$
$$\overline{13x = 117}$$
$$x = 9$$

From (1)

$3(9) - y = 23$

$y = 27 - 23$

$y = 4$

Solution 3:

The given pair of linear equations are

$$\frac{5y}{2} - \frac{x}{3} = 8$$

$$\Rightarrow -\frac{x}{3} + \frac{5y}{2} = 8 \ldots\ldots\ldots\ldots(i) \quad \text{[On simplifying]}$$

$$\frac{y}{2} + \frac{5x}{3} = 12$$

$$\Rightarrow \frac{5x}{3} + \frac{y}{2} = 12 \ldots\ldots\ldots\ldots(ii) \quad \text{[On simplifying]}$$

Multiply equation (i) by 5, we get:

$$-\frac{5x}{3} + \frac{25y}{2} = 40$$

$$\frac{5x}{3} + \frac{y}{2} = 12 \qquad \text{[Equation } (ii)\text{]}$$

$$+ \quad + \quad + \qquad \text{[Adding]}$$

$$\frac{26y}{2} = 52$$

$$\Rightarrow 13y = 52$$

$$\Rightarrow y = 4$$

Substituting $y = 4$ in equation (i), we get

$$-\frac{x}{3} + \frac{5(4)}{2} = 8$$

$$\Rightarrow -\frac{x}{3} = 8 - 10$$

$$\Rightarrow x = 6$$

$\therefore$ Solution is $x = 6$ and $y = 4$.

Solution 4:

$$\frac{1}{5}(x-2)=\frac{1}{4}(1-y) \Rightarrow \quad 4x+5y=13 \quad \ldots(1)$$

$$26x+3y=-4 \quad \ldots(2)$$

Multiplying equation no. (1) by 3 and(2) by 5.

$$12x+15y=39 \quad \ldots(3)$$

$$130x+15y=-20$$

$$- \quad - \quad +$$

$$-115x=59$$

$$x=-\frac{59}{118}$$

$$x=-\frac{1}{2}$$

From (1)

$$4\left(-\frac{1}{2}\right)+5y=13$$

$$5y=13+2$$

$$y=3$$

Solution 5:

y = 2x - 6

y = 0

$$\Rightarrow 2x-y=6 \quad \ldots(1)$$

$$y=2 \quad \ldots(2)$$

$$2x=6$$

x = 3y = 0

Solution 6:

The given pair of linear equations are

$\frac{x-y}{6} = 2(4-x)$

$\Rightarrow 13x - y = 48................(i)$ [On simplifying]

2x + y = 3(x - 4)

$\Rightarrow x - y = 12..................(ii)$ [On simplifying]

Multiply equation $(ii)$by 13, we get:

$13x - 13y = 156$

$13x - y = 48$ [Equation (i)]

$- \quad + \quad -$ [Subtracting]

$-12y = 108$

$\Rightarrow y = -9$

Substituting $y = -9$ in equation (i), we get

$13x - (-9) = 48$

$\Rightarrow 13x = 39$

$\Rightarrow x = 3$

$\therefore$ Solution is $x = 3$ and $y = -9$.

Solution 7:

3 - (x - 5) = 4 + 2

2(x + y) = 4 - 3y

$\Rightarrow -x - y = -6$

$\Rightarrow x + y = 6...(1)$

$2x + 5y = 4...(2)$

Multiplying equation no. (1) by 2.

$2x + 2y = 12$

$2x + 5y = 4$

$- \quad - \quad -$

$-3y = 8 \quad \Rightarrow y = \frac{-8}{3}$

From (1)

$x - \frac{8}{3} = 6 \qquad \Rightarrow x = \frac{26}{3}$

Solution 8:

2x -3y - 3 = 0

$$\frac{2x}{3} + 4y + \frac{1}{2} = 0$$

$\Rightarrow$ 2x - 3y = 3...(1)

$\Rightarrow$ 4x + 24y = -3...(2)

Multiplying equation no. (1) by 8.

$$16x - 24y = 24 \qquad \ldots(3)$$

$$\underline{4x + 24y = -3}$$

$$20x = 21 \quad \Rightarrow x = \frac{21}{20}$$

From (1)

$$2\left(\frac{21}{20}\right) - 3y = 3$$

$$-3y = 3 - \frac{21}{10} \Rightarrow y = \frac{-3}{10}$$

Solution 9:

13x + 11y = 70...(1)

11x+ 13y = 74...(2)

Adding(1) and (2)

24x + 24y = 144

x + y = 6...(3)

subtracting (2) from (1)

2x - 2y = -4

x - y = -2...(4)

x + y = 6...(3)

$$2x = 4 \qquad \Rightarrow x = 2$$

From (3)

2 + y = 6 $\Rightarrow$ y = 4

Solution 10:

$41x + 53y = 135 \ldots (1)$
$53x + 41y = 147 \ldots (2)$
Adding (1) and (2)
$94x + 94y = 282$
$x + y = 3 \ldots (3)$

Subtracting (2) from (1)

$$-12x + 12y = -12$$
$$-x + y = -1 \quad \ldots (4)$$
$$x + y = 3$$
$$2y = 2 \quad \Rightarrow y = 1$$

From (3)
$x + 1 = 3 \Rightarrow x = 2$

Solution 11:

$2x + y = 23 \ldots (1)$

$4x - y = 19 \ldots (2)$

Adding equation (1) and (2) we get,
$2x + y = 23$
$4x - y = 19$
$6x = 42 \quad \Rightarrow x = 7$

From (1)
$2(7) + y = 23$
$y = 23 - 14$
$\Rightarrow y = 9$
$\therefore x - 3y = 7 - 3(9) = -20$

And $5y - 2x = 5(9) - 2(7) = 45 - 14 = 31$

Solution 12:

10 y = 7x - 4

-7x + 10y = -4...(1)

12x + 18y = 1...(2)

Multiplying equation no. (1) by 12 and (2) by 7.

$$\begin{array}{r} -84x + 120y = -48 \quad \ldots(3) \\ 84x + 126y = 7 \\ \hline 246y = -41 \end{array} \Rightarrow y = \frac{-1}{6}$$

From (1)

$-7x + 10\left(\frac{-1}{6}\right) = -4$

$-7x = -4 + \frac{5}{3} \Rightarrow x = \frac{1}{3}$

$\therefore 4\left(\frac{1}{3}\right) + 6\left(\frac{-1}{6}\right) = \frac{1}{3}$ and $8y - x = 8\left(\frac{-1}{6}\right) - \frac{1}{3} = \frac{-5}{3}$

Solution 13:

(i)

The given pair of linear equations are

$\frac{y+7}{5} = \frac{2y-x}{4} + 3x - 5$

$\Rightarrow 55x + 6y = 128 \ldots\ldots\ldots\ldots\ldots(i)$ [On simplifying]

$\frac{7-5x}{2} + \frac{3-4y}{6} = 5y - 18$

$\Rightarrow 15x + 34y = 132 \ldots\ldots\ldots\ldots\ldots(ii)$ [On simplifying]

Multiply equation (i) by 3 and equation (ii) by 11, we get:

$$\begin{array}{r} 165x + 18y = 384 \\ 165x + 374y = 1452 \\ - \quad - \quad - \\ \hline -356y = -1068 \end{array} \quad \text{[Subtracting]}$$

$\Rightarrow y = 3$

Substituting $y = 3$ in equation (i), we get

$55x + 6(3) = 128$

$\Rightarrow 55x = 110$

$\Rightarrow x = 2$

$\therefore$ Solution is $x = 2$ and $y = 3$.

(ii)

The given pair of linear equations are

$$4x = 17 - \frac{x-y}{8}$$

$\Rightarrow 33x - y = 136 \ldots\ldots\ldots\ldots\ldots (i)$ [On simplifying]

$$2y + x = 2 + \frac{5y+2}{3}$$

$\Rightarrow 3x + y = 8 \ldots\ldots\ldots\ldots\ldots\ldots (ii)$ [On simplifying]

Multiply equation (ii) by 11, we get:

$33x + 11y = 88$

$33x - y = 136$ [Equation (i)]

$- \quad + \quad -$ [Subtracting]

$12y = -48$

$\Rightarrow y = -4$

Substituting $y = -4$ in equation (i), we get:

$33x - (-4) = 136$

$\Rightarrow 33x = 132$

$\Rightarrow x = 4$

$\therefore$ Solution is $x = 4$ and $y = -4$.

Solution 14:

Let x = 2 and y = 1 be a solution of the equation

$2x + 3y = m$.

$\Rightarrow 2(2) + 3(1) = m$

$\Rightarrow 4 + 3 = m$

$\Rightarrow m = 7$

$\therefore$ If x = 2 and y = 1 is the solution of the equation $2x + 3y = m$ then the value of m is 7.

Solution 15:

10% of x + 20% of y = 24

$\Rightarrow 0.1x + 0.2y = 24..............(i)$ [On simplyfying]

$3x - y = 20..............(ii)$

Multiply equation (*ii*) by 0.2, we get:

$0.6x - 0.2y = 4$

$0.1x + 0.2y = 24$ [Equation (i)]

$+ \quad + \quad +$ [Adding]

$0.7x = 28$

$\Rightarrow x = 40$

Substituting $x = 40$ in equation (i), we get

$0.1(40) + 0.2y = 24$

$\Rightarrow 0.2y = 20$

$\Rightarrow y = 100$

$\therefore$ Solution is $x = 40$ and $y = 100$.

Solution 16:

The value of expression mx - ny is 3 when x = 5 and y = 6.

$\Rightarrow 5m - 6n = 3..............(i)$

The value of expression mx - ny is 8 when x = 6 and y = 5.

$\Rightarrow 6m - 5n = 8..............(ii)$

Multiply equation (*i*) by 6 and equation (*ii*) by 5, we get:

$30m - 36n = 18$ [Equation (i)]

$30m - 25n = 40$ [Equation (ii)]

$- \quad + \quad -$ [Subtracting]

$-11n = -22$

$\Rightarrow n = 2$

Substituting $n = 2$ in equation (i), we get

$5m - 6(2) = 3$

$\Rightarrow 5m = 15$

$\Rightarrow m = 3$

$\therefore$ Solution is $m = 3$ and $n = 2$.

Solution 17:

$11(x-5)+10(y-2)+54=0 \quad \text{(given)}$

$\Rightarrow 11x-55+10y-20+54=0$

$\Rightarrow 11x+10y-21=0$

$\Rightarrow 11x+10y=21 \quad ....(1)$

$7(2x-1)+9(3y-1)=25 \quad \text{(given)}$

$\Rightarrow 14x-7+27y-9=25$

$\Rightarrow 14x+27y-16=25$

$\Rightarrow 14x+27y=41 \quad ....(2)$

Multiplying equation (1) by 27 and equation (2) by 10, we get

$297x+270y=567 \quad ....(3)$

$140x+270y=410 \quad ....(4)$

Subtracting equation (4) from equation (3), we get

$157x=157$

$\Rightarrow x=1$

Substituting $x = 1$ in equation (1), we get

$11 \times 1+10y=21$

$\Rightarrow 10y=10$

$\Rightarrow y=1$

$\therefore$ Solution set is $x = 1$ and $y = 1$.

Solution 18:

$\frac{7+x}{5}-\frac{2x-y}{4}=3y-5 \quad \text{(given)}$

$\Rightarrow 4(7+x)-5(2x-y)=20(3y-5)$

$\Rightarrow 28+4x-10x+5y=60y-100$

$\Rightarrow -6x-55y=-128 \quad ....(1)$

$\frac{5y-7}{2}+\frac{4x-3}{6}=18-5x \quad \text{(given)}$

$\Rightarrow 3(5y-7)+4x-3=6(18-5x)$

$\Rightarrow 15y-21+4x-3=108-30x$

$\Rightarrow 34x+15y=132 \quad ....(2)$

Multiplying equation (1) by 34 and equation (2) by 6, we get

$-204x-1870y=-4352 \quad ....(3)$

$204x+90y=792 \quad ....(4)$

Adding equations (3) and (4), we get

$-1780y=-3560$

$\Rightarrow y=2$

Substituting $y = 2$ in equation (1), we get

$-6x-55 \times 2=-128$

$\Rightarrow -6x-110=-128$

$\Rightarrow -6x=-18$

$\Rightarrow x=3$

$\therefore$ Solution is $x = 3$ and $y = 2$

Solution 19:

$4x + \frac{x-y}{8} = 17$ (given)

$\Rightarrow 32x + x - y = 136$

$\Rightarrow 33x - y = 136$ ....(1)

$2y + x - \frac{5y+2}{3} = 2$ (given)

$\Rightarrow 6y + 3x - 5y - 2 = 6$

$\Rightarrow 3x + y = 8$ ....(2)

Adding equations (1) and (2), we get

$36x = 144$

$\Rightarrow x = 4$

Substituting x = 4 in equation (2), we get

$3 \times 4 + y = 8$

$\Rightarrow 12 + y = 8$

$\Rightarrow y = -4$

$\therefore$ Solution is x = 4 and y = −4

Exercise 6(C)

Solution 1:

Given equations are $4x + 3y = 17$ and $3x - 4y + 6 = 0$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = 4, b_1 = 3, c_1 = -17$ and $a_2 = 3, b_2 = -4, c_2 = 6$

Now, $x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ and $y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

$\Rightarrow x = \frac{3 \times 6 - (-4) \times (-17)}{4 \times (-4) - 3 \times 3}$ and $y = \frac{-17 \times 3 - 6 \times 4}{4 \times (-4) - 3 \times 3}$

$\Rightarrow x = \frac{18 - 68}{-16 - 9}$ and $y = \frac{-51 - 24}{-16 - 9}$

$\Rightarrow x = \frac{-50}{-25}$ and $y = \frac{-75}{-25}$

$\Rightarrow x = 2$ and $y = 3$

Solution 2:

Given equations are 3x + 4y = 11 and 2x + 3y = 8

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = 3, b_1 = 4, c_1 = -11$ and $a_2 = 2, b_2 = 3, c_2 = -8$

Now, $x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ and $y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

$\Rightarrow x = \frac{4 \times (-8) - 3 \times (-11)}{3 \times 3 - 2 \times 4}$ and $y = \frac{-11 \times 2 - (-8) \times 3}{3 \times 3 - 2 \times 4}$

$\Rightarrow x = \frac{-32 + 33}{9 - 8}$ and $y = \frac{-22 + 24}{9 - 8}$

$\Rightarrow x = 1$ and $y = 2$

Solution 3:

Given equations are 6x + 7y − 11 = 0 and 5x + 2y = 13

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = 6, b_1 = 7, c_1 = -11$ and $a_2 = 5, b_2 = 2, c_2 = -13$

Now, $x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ and $y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

$\Rightarrow x = \frac{7 \times (-13) - 2 \times (-11)}{6 \times 2 - 5 \times 7}$ and $y = \frac{-11 \times 5 - (-13) \times 6}{6 \times 2 - 5 \times 7}$

$\Rightarrow x = \frac{-91 + 22}{12 - 35}$ and $y = \frac{-55 + 78}{12 - 35}$

$\Rightarrow x = \frac{-69}{-23}$ and $y = \frac{23}{-23}$

$\Rightarrow x = 3$ and $y = -1$

Solution 4:

Given equations are 5x + 4y + 14 = 0 and 3x = −10 − 4y

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = 5, b_1 = 4, c_1 = 14$ and $a_2 = 3, b_2 = 4, c_2 = 10$

Now, $x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ and $y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

$\Rightarrow x = \frac{4 \times 10 - 4 \times 14}{5 \times 4 - 3 \times 4}$ and $y = \frac{14 \times 3 - 10 \times 5}{5 \times 4 - 3 \times 4}$

$\Rightarrow x = \frac{40 - 56}{20 - 12}$ and $y = \frac{42 - 50}{20 - 12}$

$\Rightarrow x = \frac{-16}{8}$ and $y = \frac{-8}{8}$

$\Rightarrow x = -2$ and $y = -1$

Solution 5:

Given equations are $x - y + 2 = 0$ and $7x + 9y = 130$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = 1, b_1 = -1, c_1 = 2$ and $a_2 = 7, b_2 = 9, c_2 = -130$

Now, $x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ and $y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

$\Rightarrow x = \frac{-1 \times (-130) - 9 \times 2}{1 \times 9 - 7 \times (-1)}$ and $y = \frac{2 \times 7 - (-130) \times 1}{1 \times 9 - 7 \times (-1)}$

$\Rightarrow x = \frac{130 - 18}{9 + 7}$ and $y = \frac{14 + 130}{9 + 7}$

$\Rightarrow x = \frac{112}{16}$ and $y = \frac{144}{16}$

$\Rightarrow x = 7$ and $y = 9$

Solution 6:

Given equations are $4x - y = 5$ and $5y - 4x = 7$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = 4, b_1 = -1, c_1 = -5$ and $a_2 = -4, b_2 = 5, c_2 = -7$

Now, $x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ and $y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

$\Rightarrow x = \frac{-1 \times (-7) - 5 \times (-5)}{4 \times 5 - (-4) \times (-1)}$ and $y = \frac{(-5) \times (-4) - (-7) \times 4}{4 \times 5 - (-4) \times (-1)}$

$\Rightarrow x = \frac{7 + 25}{20 - 4}$ and $y = \frac{20 + 28}{20 - 4}$

$\Rightarrow x = \frac{32}{16}$ and $y = \frac{48}{16}$

$\Rightarrow x = 2$ and $y = 3$

Solution 7:

Given equations are $4x - 3y = 0$ and $2x + 3y = 18$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = 4, b_1 = -3, c_1 = 0$ and $a_2 = 2, b_2 = 3, c_2 = -18$

Now, $x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ and $y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

$\Rightarrow x = \frac{-3 \times (-18) - 3 \times 0}{4 \times 3 - 2 \times (-3)}$ and $y = \frac{0 \times 2 - (-18) \times 4}{4 \times 3 - 2 \times (-3)}$

$\Rightarrow x = \frac{54 - 0}{12 + 6}$ and $y = \frac{0 + 72}{12 + 6}$

$\Rightarrow x = \frac{54}{18}$ and $y = \frac{72}{18}$

$\Rightarrow x = 3$ and $y = 4$

Solution 8:

Given equations are $8x + 5y = 9$ and $3x + 2y = 4$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = 8, b_1 = 5, c_1 = -9$ and $a_2 = 3, b_2 = 2, c_2 = -4$

Now, $x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ and $y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

$\Rightarrow x = \frac{5 \times (-4) - 2 \times (-9)}{8 \times 2 - 3 \times 5}$ and $y = \frac{-9 \times 3 - (-4) \times 8}{8 \times 2 - 3 \times 5}$

$\Rightarrow x = \frac{-20 + 18}{16 - 15}$ and $y = \frac{-27 + 32}{16 - 15}$

$\Rightarrow x = \frac{-2}{1}$ and $y = \frac{5}{1}$

$\Rightarrow x = -2$ and $y = 5$

Solution 9:

Given equations are $4x - 3y - 11 = 0$ and $6x + 7y - 5 = 0$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = 4, b_1 = -3, c_1 = -11$ and $a_2 = 6, b_2 = 7, c_2 = -5$

Now, $x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ and $y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

$\Rightarrow x = \frac{-3 \times (-5) - 7 \times (-11)}{4 \times 7 - 6 \times (-3)}$ and $y = \frac{-11 \times 6 - (-5) \times 4}{4 \times 7 - 6 \times (-3)}$

$\Rightarrow x = \frac{15 + 77}{28 + 18}$ and $y = \frac{-66 + 20}{28 + 18}$

$\Rightarrow x = \frac{92}{46}$ and $y = \frac{-46}{46}$

$\Rightarrow x = 2$ and $y = -1$

Solution 10:

Given equations are $4x + 6y = 15$ and $3x - 4y = 7$

Comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we have

$a_1 = 4,\ b_1 = 6,\ c_1 = -15$ and $a_2 = 3,\ b_2 = -4,\ c_2 = -7$

Now, $x = \dfrac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ and $y = \dfrac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

$\Rightarrow x = \dfrac{6 \times (-7) - (-4) \times (-15)}{4 \times (-4) - 3 \times 6}$ and $y = \dfrac{-15 \times 3 - (-7) \times 4}{4 \times (-4) - 3 \times 6}$

$\Rightarrow x = \dfrac{-42 - 60}{-16 - 18}$ and $y = \dfrac{-45 + 28}{-16 - 18}$

$\Rightarrow x = \dfrac{-102}{-34}$ and $y = \dfrac{-17}{-34}$

$\Rightarrow x = 3$ and $y = \dfrac{1}{2}$

Exercise 6(D)

Solution 1:

$$\frac{9}{x} - \frac{4}{y} = 8 \qquad \ldots(1)$$

$$\frac{13}{x} + \frac{7}{y} = 101 \qquad \ldots(2)$$

Multiplying equation no. (1) by 7 and (2) by 4.

$$\frac{63}{x} - \frac{28}{y} = 56 \qquad \ldots(3)$$

$$\frac{52}{x} + \frac{28}{y} = 404 \qquad \ldots(4)$$

$$\frac{115}{x} = 460$$

$$x = \frac{115}{460} \Rightarrow x = \frac{1}{4}$$

From (1)

$$9 \times \left(\frac{4}{1}\right) - \frac{4}{y} = 8$$

$$-\frac{4}{y} = -28 \Rightarrow y = \frac{1}{7}$$

Solution 2:

$\frac{3}{x}+\frac{2}{y}=10$ ....(i)

$\frac{9}{x}-\frac{7}{y}=10.5$ ....(ii)

Multiplying equation (i) by 3, we get

$\frac{9}{x}+\frac{6}{y}=30$ ....(iii)

Subtracting (ii) from (iii), we get

$\frac{13}{y}=19.5$

$\Rightarrow y=\frac{13}{19.5}=\frac{2}{3}$

From (i),

$\frac{3}{x}+\frac{2\times 3}{2}=10$

$\Rightarrow \frac{3}{x}+3=10$

$\Rightarrow \frac{3}{x}=7$

$\Rightarrow x=\frac{3}{7}$

Solution 3:

$5x+\frac{8}{y}=19$ ....(i)

$3x-\frac{4}{y}=7$ ....(ii)

Multiplying equation (ii) by 2, we get

$6x-\frac{8}{y}=14$ ....(iii)

Adding (i) and (iii), we get

$11x=33$

$\Rightarrow x=3$

Substituting $x=3$ in equation 91), we get

$5(3)+\frac{8}{y}=19$

$\Rightarrow \frac{8}{y}=19-15$

$\Rightarrow y=\frac{8}{4}=2$

Solution 4:

$4x + \frac{6}{y} = 15$ ....(i)

$3x - \frac{4}{y} = 7$ ....(ii)

Multiplying (i) by 4 and (ii) by 6

$16x + \frac{24}{y} = 60$ ....(iii)

$18x - \frac{24}{y} = 42$ ....(iv)

Adding (iii) and (iv), we get

$34x = 102$

$\Rightarrow x = 3$

Substituting $x = 3$ in (i), we get

$4(3) + \frac{6}{y} = 15$

$\Rightarrow \frac{6}{y} = 15 - 12$

$\Rightarrow y = \frac{6}{3} = 2$

Now, $y = ax - 2$

$\Rightarrow 2 = a(3) - 2$

$\Rightarrow 2 = 3a - 2$

$\Rightarrow 3a = 4$

$\Rightarrow a = \frac{4}{3} = 1\frac{1}{3}$

Solution 5:

$\frac{3}{x} - \frac{2}{y} = 0$ ...(1)

$\frac{2}{x} + \frac{5}{y} = 19$ ...(2)

Multiplying equation no. (1) by 5 and (2) by 2.

$\frac{15}{x} - \frac{10}{y} = 0$ ...(3)

$\frac{4}{x} + \frac{10}{y} = 38$ ....(4)

$\frac{19}{x} = 38$ $\Rightarrow x = \frac{1}{2}$

From (1) $3\left(\frac{1}{2}\right) - \frac{2}{y} = 0$ $\Rightarrow y = \frac{1}{3}$

$\therefore y = ax + 3$

$\frac{1}{3} = a\left(\frac{1}{2}\right) + 3$

$\frac{a}{2} = \frac{-8}{3} \Rightarrow a = \frac{-16}{3}$

Solution 6:

(i)

$$\frac{20}{x+y}+\frac{3}{x-y}=7 \quad ...(1)$$

$$\frac{8}{x+y}-\frac{15}{x+y}=5 \quad ...(2)$$

Multiplying equation no. (1) by 8 and (2) by 3.

$$\frac{160}{x+y}+\frac{24}{x-y}=56 \quad ...(3)$$

$$\frac{-45}{x+y}+\frac{24}{x-y}=15 \quad ...(4)$$

$$\underline{-\qquad -\qquad\qquad -}$$

$$\frac{205}{x+y}=41$$

$x+y=5 \quad ...(5)$

From (1)

$$\frac{20}{5}+\frac{3}{x-y}=7$$

$$\frac{3}{x-y}=3$$

$x-y=1 \quad ...(6)$

$x+y=5 \quad ...(5)$

$\underline{x-y=1} \quad ...(6)$

$2x = 6$

$x = 3$

from (5)

$3+y=5 \Rightarrow y=2$

(ii)

Let $a = 3x + 4y$ and $b = 3x - 2y$

$$\therefore \frac{34}{3x+4y} + \frac{15}{3x-2y} = 5$$

$$\Rightarrow \frac{34}{a} + \frac{15}{b} = 5 \text{..........(i)}$$

$$\frac{25}{3x-2y} - \frac{8.50}{3x+4y} = 4.5$$

$$\Rightarrow -\frac{8.50}{a} + \frac{25}{b} = 4.5 \text{..........(ii)}$$

Multiply equation (ii)by 4, we get :

$$-\frac{34}{a} + \frac{100}{b} = 18$$

$$\frac{34}{a} + \frac{15}{b} = 5 \qquad \text{[Equation (i)]}$$

$$+ \quad + \quad + \qquad \text{[Adding]}$$

$$\frac{115}{b} = 23$$

$$\Rightarrow b = 5$$

$$\Rightarrow 3x - 2y = 5 \text{..........(iii)}$$

Substituting $b = 5$ in equation (i), we get

$$\frac{34}{a} + \frac{15}{5} = 5$$

$$\Rightarrow 2a = 34$$

$$\Rightarrow a = 17$$

$$\Rightarrow 3x + 4y = 17 \text{..........(iv)}$$

Subtracting equation (iv)from equation (iii), we get::

$$3x - 2y = 5$$

$$3x + 4y = 17$$

$$- \quad - \quad -$$

$$-6y = -12$$

$$\Rightarrow y = 2$$

SubStituting $y = 2$ in equation (iii), we get

$$3x - 2(2) = 5$$

$$\Rightarrow 3x = 9$$

$$\Rightarrow x = 3$$

$\therefore$ Solution is $x = 3$ and $y = 2$.

Solution 7:

(i)

$$x + y = 2xy \qquad \ldots(1)$$

$$\underline{x - y = 6xy} \qquad \ldots(2)$$

$$2x = 8xy$$

$$2 = 8y$$

$$y = \frac{1}{4}$$

From (1)

$$x + \frac{1}{4} = 2x\left(\frac{1}{4}\right)$$

$$\frac{1}{2}x = \frac{-1}{4}$$

$$x = \frac{-1}{2}$$

(ii)

x + y = 7xy ...(1)

2x - 3 = -xy ...(2)

Multiplying equation no. (1) by 3.

$$3x + 3y = 21xy \qquad \ldots(3)$$

$$\underline{2x - 3y = -xy} \qquad \ldots(4)$$

$$5x = 20xy$$

$$y = \frac{1}{4}$$

From (1)

$$x + \frac{1}{4} = 7x\left(\frac{1}{4}\right)$$

$$\frac{1}{4} = \frac{3}{4}x$$

$$x = \frac{1}{3}$$

Solution 8:

Given equations are $\frac{a}{x} - \frac{b}{y} = 0$ and $\frac{ab^2}{x} + \frac{a^2b}{y} = a^2 + b^2$

Taking $\frac{1}{x} = u$ and $\frac{1}{y} = v$, the above system of equations become

$au - bv + 0 = 0$

$ab^2u + a^2bv - (a^2 + b^2) = 0$

By cross-multiplication, we have

$$\frac{u}{-b \times [-(a^2 + b^2)] - a^2b \times 0} = \frac{-v}{a \times [-(a^2 + b^2)] - ab^2 \times 0} = \frac{1}{a \times a^2b - ab^2 \times (-b)}$$

$$\Rightarrow \frac{u}{b(a^2 + b^2)} = \frac{-v}{-a(a^2 + b^2)} = \frac{1}{a^3b + ab^3}$$

$$\Rightarrow \frac{u}{b(a^2 + b^2)} = \frac{v}{a(a^2 + b^2)} = \frac{1}{ab(a^2 + b^2)}$$

$$\Rightarrow u = \frac{b(a^2 + b^2)}{ab(a^2 + b^2)} \quad \text{and} \quad v = \frac{a(a^2 + b^2)}{ab(a^2 + b^2)}$$

$$\Rightarrow u = \frac{1}{a} \text{ and } v = \frac{1}{b}$$

$$\Rightarrow \frac{1}{x} = \frac{1}{a} \text{ and } \frac{1}{y} = \frac{1}{b}$$

$$\Rightarrow x = a \text{ and } y = b$$

Solution 9:

$$\frac{2xy}{x + y} = \frac{3}{2}$$

$$\Rightarrow \frac{x + y}{xy} = \frac{4}{3}$$

$$\Rightarrow \frac{1}{x} + \frac{1}{y} = \frac{4}{3} \quad \ldots(1)$$

$$\frac{xy}{2x - y} = -\frac{3}{10}$$

$$\Rightarrow \frac{2x - y}{xy} = -\frac{10}{3}$$

$$\Rightarrow -\frac{1}{x} + \frac{2}{y} = -\frac{10}{3} \quad \ldots(2)$$

Let $\frac{1}{x} = u$ and $\frac{1}{y} = v$

Then, equations (1) and (2) become

$u + v = \frac{4}{3}$ and $-u + 2v = -\frac{10}{3}$

$\Rightarrow 3u + 3v = 4$ and $-3u + 6v = -10$

Adding, we have

$9v = -6$

$$\Rightarrow v = -\frac{6}{9} = -\frac{2}{3}$$

$$\Rightarrow \frac{1}{y} = -\frac{2}{3} \Rightarrow y = -\frac{3}{2}$$

Substituting $y = -\frac{3}{2}$ in (1), we have

$$\frac{1}{x} - \frac{2}{3} = \frac{4}{3}$$

$$\Rightarrow \frac{1}{x} = \frac{6}{3} = 2$$

$$\Rightarrow x = \frac{1}{2}$$

Solution 10:

Given equations are $\frac{3}{2x} + \frac{2}{3y} = -\frac{1}{3}$ and $\frac{3}{4x} + \frac{1}{2y} = -\frac{1}{8}$

Let $\frac{1}{x} = u$ and $\frac{1}{y} = v$

Then, the system of equations become

$\frac{3}{2}u + \frac{2}{3}v = -\frac{1}{3}$ and $\frac{3}{4}u + \frac{1}{2}v = -\frac{1}{8}$

$\Rightarrow \frac{9u + 4v}{6} = -\frac{1}{3}$ and $\frac{3u + 2v}{4} = -\frac{1}{8}$

$\Rightarrow 27u + 12v = -6$ and $24u + 16v = -4$

$\Rightarrow 27u + 12v + 6 = 0$ and $24u + 16v + 4 = 0$

$\Rightarrow \frac{u}{12 \times 4 - 16 \times 6} = \frac{-v}{27 \times 4 - 24 \times 6} = \frac{1}{27 \times 16 - 24 \times 12}$

$\Rightarrow \frac{u}{48 - 96} = \frac{-v}{108 - 144} = \frac{1}{432 - 288}$

$\Rightarrow \frac{u}{-48} = \frac{-v}{-36} = \frac{1}{144}$

$\Rightarrow \frac{u}{-48} = \frac{v}{36} = \frac{1}{144}$

$\Rightarrow u = \frac{-48}{144} = -\frac{1}{3}$ and $v = \frac{36}{144} = \frac{1}{4}$

$\Rightarrow \frac{1}{x} = -\frac{1}{3}$ and $\frac{1}{y} = \frac{1}{4}$

$\Rightarrow x = -3$ and $y = 4$

Exercise 6(E)

Solution 1:

Let the two numbers be x and y
According to the question,

$$\frac{x}{y} = \frac{2}{3}$$

3x - 2y = 0 ...(1)

Also, $\frac{x-2}{y-8} = \frac{3}{2}$

2x - 3y = -20 ...(2)

Multiplying equation no. (1) by 2 and (2) by 3and substracting

$$\begin{array}{l} 6x - 4y = 0 \\ 6x - 9y = -60 \\ \underline{- \quad + \qquad +} \\ \quad 5y = 60 \end{array}$$

$$y = 12$$

From (1), we get

3x - 2(12) = 0

$$x = \frac{24}{3}$$

x = 8

Thus, the numbers are 8 and 12.

Solution 2:

Let the smaller number be x
and the larger number bey.
According to the question,

$$\frac{x}{y} = \frac{4}{7}$$

7x -; 4y = 0...(1)
and,3y + 2x = 59...(2)
Multiplying equation no. (1) by 3 and (2) by 4.and adding them

$$\begin{array}{ll} 21x - 12y = 0 & \ldots(3) \\ \underline{8x + 12y = 236} & \ldots(4) \\ \quad 29x = 236 & \end{array}$$

$$x = \frac{236}{29}$$

From (1)

$$7\left(\frac{236}{29}\right) = 4y$$

$$y = 7\left(\frac{59}{29}\right)$$

$$y = \frac{413}{29}$$

Hence, the number are $\frac{236}{29}$ and $\frac{413}{29}$.

Solution 3:

Let x be the greater number and y be the smaller number.

When the greater of the two numbers increased by 1

divides the sum of the numbers, the result is $\frac{3}{2}$.

$\Rightarrow \frac{x+y}{(x+1)} = \frac{3}{2}$

$\Rightarrow 2x + 2y = 3(x+1)$

$\Rightarrow x - 2y = -3 \ldots\ldots\ldots\ldots\ldots (i)$

When the difference of these number is divided by the smaller,

the result is $\frac{1}{2}$.

$\Rightarrow \frac{x-y}{y} = \frac{1}{2}$

$\Rightarrow 2x - 2y = y$

$\Rightarrow 2x - 3y = 0 \ldots\ldots\ldots\ldots\ldots (ii)$

Multiply equation (i) by 2, we get:

$2x - 4y = -6$

$2x - 3y = 0$ [Equation (ii)]

$- \quad + \quad -$ [Subtracting]

$-y = -6$

$\Rightarrow y = 6$

Substituting $y = 6$ in equation (i), we get

$x - 2(6) = -3$

$\Rightarrow x = 9$

$\therefore$ 9 is the greater number and 6 is the smaller number.

Solution 4:

Let the common multiple between the numbers be x.

So, the numbers are 4x and 5x.

According to the question,

$$\frac{4x-30}{5x-30}=\frac{1}{2}$$

$\Rightarrow 8x - 60 = 5x - 30$

$\Rightarrow 3x = 30$

$\Rightarrow x = 10$

So, $4x = 4(10) = 40$ and $5x = 5(10) = 50$

Thus, the numbers are 40 and 50.

Solution 5:

Let the numerator and denominator a fraction be x and y respectively.

According to the question,

$$\frac{x+2}{y-1}=\frac{2}{3}$$

$3x - 2y = -8$...(1)

And,

$$\frac{x+1}{y+2}=\frac{1}{3}$$

$3x - y = -1 \qquad \ldots(2)$

Now subtracting,

$$\begin{array}{rl} 3x - y = -1 & \ldots(2) \\ 3x - 2y = -8 & \ldots(1) \\ - \quad + \qquad + & \\ \hline y = 7 & \end{array}$$

From (1),

$3x - 2(7) = -8$

$3x = -8 + 14$

$x = 2$

Required fraction = $\frac{2}{7}$

Solution 6:

Let the numerator and denominator of a fraction be x and y respectively .Then the fraction will be $\frac{x}{y}$

According to the question,

$x + y = 7$...(1)

$5y - 4x = 8$...(2)

Multiplying equation no. (1) by 4 and add with (2),

$$\begin{array}{r} 4x + 4y = 28 \quad ..(3) \\ -4x + 5y = 8 \\ \hline 9y = 36 \end{array}$$

$y = 4$

From (1)

$x + 4 = 7$

$x = 3$

Required fraction = $\frac{3}{4}$

Solution 7:

Let the numerator of the fraction be x and the denominator be y.

So, the fraction is $\frac{x}{y}$.

According to the question,

$\frac{2x}{y+1} = 1 \Rightarrow 2x = y + 1 \Rightarrow 2x - y = 1$...(i)

and $\frac{x+4}{2y} = \frac{1}{2} \Rightarrow 2x + 8 = 2y \Rightarrow 2x - 2y = -8$...(ii)

Solving equations (i) and (ii), we get

$y = 9$

Putting the value of y in (i), we get

$2x - (9) = 1 \Rightarrow 2x = 1 + 9 \Rightarrow x = 5$

So, the fraction is $\frac{5}{9}$.

Solution 8:

Let the numerator of the fraction be x and denominator of the fraction be y.

Then, the fraction = $\frac{x}{y}$

According to given condition, we have

$$\frac{x-5}{y-3}=\frac{1}{2}$$

$\Rightarrow 2x-10=y-3$

$\Rightarrow 2x-y=7$ ....(i)

And,

$x+5=y$

$\Rightarrow x-y=-5$ ....(ii)

Subtracting (ii) from (i), we get

$x=12$

$\Rightarrow y=x+5=12+5=17$

hence, the fraction is $\frac{12}{17}$.

Solution 9:

Let the numerator of the fraction be x and denominator of the fraction be y.

Then, the fraction = $\frac{x}{y}$

According to given condition, we have

$$\frac{x-5}{y-3}=\frac{1}{2}$$

$\Rightarrow 2x-10=y-3$

$\Rightarrow 2x-y=7$ ....(i)

And,

$x+5=y$

$\Rightarrow x-y=-5$ ....(ii)

Subtracting (ii) from (i), we get

$x=12$

$\Rightarrow y=x+5=12+5=17$

hence, the fraction is $\frac{12}{17}$.

Solution 10:

Let the digit at unit's place be x and the digit at ten's place be y.
Required no. = 10y + x
If the digit's are reversed
Reversed no. = 10x + y
According to the question,
x + y = 7...(1)
and,
10x + y - 2 = 2(10y + x).
8x - 19y = 2...(2)
Multiplying equation no. (1) by 19.

$$19x + 19y = 133 \qquad \dots(3)$$

Now adding equation(2) and (3)

$$19x + 19y = 133 \qquad \dots(3)$$
$$8x - 19y = 2 \qquad \dots(2)$$
$$27x = 135$$

x = 5
From (1)
5 +y = 7
y = 2
Required number is
10(2) + 5
= 25.

Solution 11:

Let the digit at unit's place be x and the digit at ten's place be y.
Required no. = 10y + x
According to the question
y = 3x ⇒ 3x - y = 0...(1)
and,10y + x + x = 32
10y + 2x = 32...(2)
Multiplying equation no. (1) by 10

$$30x - 10y = 0 \qquad \dots(3)$$

Now adding (3)and(2)

$$30x - 10y = 0 \qquad \dots(3)$$
$$2x + 10y = 32 \qquad \dots(2)$$
$$32x = 32$$
$$x = 1$$

From (1),we get
y = 3(1) = 3
Required no is
10(3) + 1 = 31

Solution 12:

Let the digit a unit's place be x and the digit at ten's place be y.
Required no. = 10y + x.
According to the question,
y - 2x = 2
-2x + y = 2...(1)
and,
(10x + y) -3 (y + x) = 5
7x - 2y = 5...(2)
Multiplying equation no. (1) by 2.

$$-4x + 2y = 4 \qquad \ldots(3)$$

Now adding (2)and(3)

$$\begin{array}{r} -4x + 2y = 4 \\ 7x - 2y = 5 \\ \hline 3x = 9 \end{array}$$

$$x = 3$$

From (1) ,we get
-2(3) + y = 2
⇒ y = 8
Required number is
10(8) + 3 = 83.

Solution 13:

Let x be the number at the ten's place
and y be the number at the unit's place.
So, the number is 10x + y.

Four times a certain two-digit number is seven times the number obtained on interchanging its digits.

$$\Rightarrow 4(10x + y) = 7(10y + x)$$

$$\Rightarrow 40x + 4y = 70y + 7x$$

$$\Rightarrow 33x - 66y = 0$$

$$\Rightarrow x - 2y = 0 \ldots\ldots\ldots\ldots\ldots(i)$$

If the difference between the digits is 4, then

$$\Rightarrow x - y = 4 \ldots\ldots\ldots\ldots\ldots(ii)$$

Subtracting equation (i) from equation (ii), we get:

$$\begin{array}{rl} x - y = 4 & \\ x - 2y = 0 & \text{[Equation } (i)\text{]} \\ - \quad + \quad & \text{[Subtracting]} \\ \hline y = 4 & \end{array}$$

Substituting $y = 4$ in equation (i), we get

$$x - 2(4) = 0$$

$$\Rightarrow x = 8$$

∴ The number is $10x + y = 10(8) + 4 = 84$

Solution 14:

Let the tens digit of the number be x and the units digit be y.

So, the number is $10x + y$.

The number obtained by interchanging the digits will be $10y + x$.

According to question, we have

$10x + y + 10y + x = 121$

$\Rightarrow 11x + 11y = 121$

$\Rightarrow 11(x + y) = 121$

$\Rightarrow x + y = 11 \quad ....(i)$

And,

$x - y = 3 \quad ....(ii)$

Adding (i) and (ii), we get

$2x = 14$

$\Rightarrow x = 7$

$\Rightarrow y = 11 - x = 11 - 7 = 4$

Hence, the number is 74.

Solution 15:

Let the tens digit of the number be x and the units digit be y.

So, the number is $10x + y$.

According to the question,

$10x + y = 8(x + y) \Rightarrow 2x = 7y...(i)$

and $10x + y = 14(x - y) + 2$ or $10x + y = 14(y - x) + 2$

$\Rightarrow 4x - 15y = -2...(ii)$ or $24x - 13y = 2...(iii)$

Solving (i) and (ii), we get

$y = 2$ and $x = 7$

Solving (i) and (iii), we get

$y = \frac{2}{71}$

This is not possible, since y is a digit and cannot be in fraction form.

So the number is 72.

Exercise 6(F)

Solution 1:

Let present age of A = x years
And present age of B = y years
According to the question,
Five years ago,
$x - 5 = 4(y - 5)$
$x - 4y = -15$...(1)
Five years later,
$x + 5 = 2(y + 5)$

$$x - 2y = 5 \quad \ldots(2)$$

Now subtracting (1)from(2)

$$\begin{array}{rl} x - 2y = 5 & \ldots(2) \\ x - 4y = -15 & \ldots(1) \\ - \;\; + \quad\;\; + & \\ \hline 2y = 20 & \end{array}$$

$$y = 10$$

From (1)
$x - 4\ (10) = -15$
$x = 25$
Present ages of A and B are 25 years and 10 years respectively.

Solution 2:

Let A's presentage be x years
and B's present age be y years
According to the question
$x = y + 20$
$x - y = 20$...(1)
Five years ago,
$x - 5 = 3(y - 5)$

$$x - 3y = -10 \quad \ldots(2)$$

Subtracting (1)from(2),

$$\begin{array}{rl} x - 3y = -10 & \ldots(2) \\ x - \;\; y = \;\; 20 & \ldots(1) \\ - \;\; + \quad\;\; - & \\ \hline -2y = -30 & \end{array}$$

$y = 15$
From (1)
$x = 15 + 20$
$x = 35$
Thus, present ages of A and B are 35 years and 15 years.

Solution 3:

Let the present age of the mother be x years
and the present age of the daughter be y year.
According to the question,
$x - 4 = 4(y - 4) \Rightarrow x - 4 = 4y - 16 \Rightarrow x - 4y = -12 \ldots (i)$
and $x + 6 = 2\frac{1}{2}(y + 6) \Rightarrow x + 6 = \frac{5}{2}y + 15 \Rightarrow x - \frac{5}{2}y = 9 \ldots (ii)$
Solving (i) and (ii), we get
$y = 14$ and $x = 44$
Hence, the present age of the mother is 44 years
and the present age of the daughter is 14 years.

Solution 4:

Let the present age of the man be x years
and let the sum of the ages of his two children be y years.
According to the question,
$x = 2y \ldots (i)$
and $x + 20 = y + 40 \ldots (ii) \ldots$ (Since he has two children)
Solving (i) and (ii), we get
$2y + 20 = y + 40 \Rightarrow y = 20$
So, $x = 2y \Rightarrow x = 40$
Hence, the present age of the man is 40 years.

Solution 5:

Let A's annual in come = Rs.x
and B's annual income = Rs. y
According to the question,

$$\frac{x}{y} = \frac{3}{4}$$

$4x - 3y = 0 \ldots (1)$

and, $\frac{x - 5000}{y - 5000} = \frac{5}{7}$

$7x - 5y = 10000 \ldots (2)$
Multiplying equation no. (1) by 7 and (2) by 4.and subtracting (4) from (3)

$$28x - 21y = 0 \quad \ldots (3)$$
$$28x - 20y = 40000 \quad \ldots (4)$$
$$- \quad + \quad -$$
$$-y = -40000$$
$$y = 40,000$$

From (1)
$4x - 3(40000) = 0$
$x = 30000$
Thus, A's income in Rs. 30,000 and B's income is Rs. 40,000.

Solution 6:

Let the no. of pass candidates be x
and the no. of fail candidates be y.
According to the question,

$$\frac{x}{y} = \frac{y}{1}$$

x - 4y = 0...(1)

and $\frac{x-20}{y-10} = \frac{5}{1}$

x - 5y = -30...(2)

$$\begin{array}{ll} x - 4y = 0 & \dots(1) \\ x - 5y = -30 & \dots(2) \\ - \quad + \qquad + & \\ \hline \qquad y = 30 & \end{array}$$

From (1)
- 4(30) = 0
x = 120
Total students appeared = x + y
= 120 + 30
= 150

Solution 7:

Let the numberof pencils with A = x
and the number of pencils with B = y.
If A gives 10 pencils to B,
y + 10 = 2(x - 10)
2x - y = 30...(1)
If B gives to pencils to A
y - 10 = x + 10

$$\begin{array}{ll} x - y = -20 & \dots(2) \\ 2x - y = 30 & \dots(1) \\ - \quad + \qquad - & \\ \hline \quad -x = -50 & \end{array}$$

$$x = 50$$

From (1)
2(50) - y = 30
y = 70
Thus, A has 50 pencils and B has 70 pencils.

Solution 8:

Let the number of adults = x
and the number of children = y
According to the question,
x + y = 1250...(1)
and 75x + 25y = 61250

$$3x + y = 2450 \quad \ldots(2)$$
$$x + y = 1250 \quad \ldots(1)$$
$$- \quad - \quad -$$
$$2x = 1200$$

x = 6000
From (1)
600 + y = 1250
y = 650
Thus, number of adults = 600
and the number of children = 650.

Solution 9:

Let the cost price of article A = Rs. x
and the cost price of articles B = Rs. y
According to the question,
(x + 5% of x) +(y + 7% of y) = 1167

$$\left(x + \frac{5}{100}x\right) + \left(y \frac{7}{100}y\right) = 1167$$
$$\frac{21x}{20} + \frac{107y}{100} = 1167$$

105x + 107y = 1167...(1)

and $\frac{107x}{100} + \frac{105y}{100} = 1165$

107x + 105y = 116500...(2)
Adding(1) and (2)
212x + 212y = 233200
x + y = 1100...(3)
subtracting (2)from (1)
-2x + 2y = 200

$$-x + y = 100 \quad \ldots(4)$$
$$x + y = 1100 \quad \ldots(3)$$
$$2y = 1200$$

y = 600
from (3)
x +600 = 1100
x = 500
Thus, cost price of article A is Rs. 500.
and that of article B is Rs. 600

Solution 10:

Let Pooja's 1 day work = $\frac{1}{x}$

and Ritu's 1 day work = $\frac{1}{y}$

According the question,

$$\frac{1}{x}+\frac{1}{y}=\frac{7}{120} \qquad \ldots(1)$$

and, $\frac{1}{x}=\frac{3}{4}\cdot\frac{1}{y}$

$$y=\frac{3}{4}x \qquad \ldots(2)$$

Using the value of y from (2) in (1)

$$\frac{1}{x}+\frac{4}{3x}=\frac{7}{120}$$

$$\frac{1}{x}\left(\frac{7}{3}\right)=\frac{7}{120}$$

$$x=40$$

From (2) $y=\frac{3}{4}(40)=30$

y = 30

Pooja will complete the work in 40 days and Ritu will complete the work in 30 days.

Exercise 6(G)

Solution 1:

Let Rohit has Rs. x

and Ajay has Rs. y

When Ajay gives Rs. 100 to Rohit

x + 100 = 2(y - 100)

x - 2y = -300...(1)

When Rohit gives Rs. 10 to Ajay

6(x-10) = y + 10

6x - y = 70...(2)

Multiplying equation no. (2) By 2.

$$12x-2y=140 \qquad \ldots(3)$$

$$x-2y=-300$$

$$-\quad+\quad+$$

$$11x=440$$

x = 40

From (1)

40 - 2y = -300

⇒ -2y = -340

⇒ y = 170

Thus, Rohit has Rs. 40

Solution 2:

Let the digits in the tens place be x and the digit in the units place be y.

$\therefore$ Number = $10x + y$

Number on reversing the digits = $10y + x$

The difference between the digits = $x - y$ or $y - x$

Given : $(10x + y) + (10y + x) = 99$

$\Rightarrow 11x + 11y = 99$

$\Rightarrow x + y = 9.....(i)$

$x - y = 3.....(ii)$

or $y - x = 3....(iii)$

On solving equations (i) and (ii), we get

$2x = 12 \Rightarrow x = 6$

So, $y = 3$

On solving equations (i) and (iii), we get

$2y = 12 \Rightarrow y = 6$

So, $x = 3$

Number $= 10x + y = 10(6) + 3 = 63$

or Number $= 10x + y = 10(3) + 6 = 36$

$\therefore$ Required number = 63 or 36.

Solution 3:

3Let the digit at ten's place be x

And the digit at unit's place be y

Required number = $10x + y$

When the digits are interchanged,

Reversed number = $10y + x$

According to the question,

$7(10x + y) = 4(10y + x)$

$66x = 33y$

$2x - y = 0...(1)$

Also,

$$\begin{array}{rl} y - x = 3 & \quad ...(2) \\ \underline{-y + 2x = 0} & \quad ...(1) \\ x = 3 & \end{array}$$

From (1) $2(3) - y = 0$

$y = 6$

Thus, Required number = $10(3) + 6 = 36$

Solution 4:

Let, the fare of ticket for station A be Rs. x
and the fare of ticket for station B be Rs. y
According, to the question
$2x + 3y = 77$....(1)
and$3x+5y = 124$...(2)
Multiplying equation no. (1) by 3 and (2) by 2.

$$\begin{array}{ll} 6x + 9y = 231 & \ldots(1) \\ 6x + 10y = 248 & \ldots(4) \\ - \quad - \qquad - & \\ \hline -y = -17 & \end{array}$$

$y = 17$
From (1) $2x + 3(17) = 77$
$2x = 77 - 51$
$2x = 26$
$x = 13$
Thus, fare for station A = Rs. 13
and, fare for station B = Rs. 17.

Solution 5:

Let x be the number at the ten's place
and y be the number at the unit's place.
So the number is $10x + y$.

The sum of digit of a two digit number is 11.
$\Rightarrow x + y = 11$...............(i)

If the digit at ten's place is increased by 5
and the digit at unit place is decreased by 5,
the digits of the number are found to be reversed.
$\Rightarrow 10(x + 5) + (y - 5) = 10y + x$
$\Rightarrow 9x - 9y = -45$
$\Rightarrow x - y = -5$...............(ii)

Subtracting equation (i) from equation (ii), we get:

$$\begin{array}{ll} x - y = -5 & \\ x + y = 11 & [\text{Equation } (i)] \\ - \quad - \qquad - & [\text{Subtracting}] \\ \hline -2y = -16 & \end{array}$$

$\Rightarrow y = 8$

Substituting $y = 8$ in equation (i), we get
$x + 8 = 11$
$\Rightarrow x = 3$

Solution 6:

Let the quantity of 90% acid solution be x litres and
The quantity of 97% acid solution be y litres
According to the question,
$x + y = 21$...(1)
and 90% of x + 97% of y = 95% of 21
$90x + 97y = 1995$...(2)
Multiplying equation no. (1) by90, we get,

$$90x + 90y = 1890 \quad \ldots(3)$$
$$90x + 97y = 1995 \quad \ldots(2)$$
$$- \quad - \quad\quad +$$
$$-7y = -105$$
$$y = 15$$

From (1)$x + 15 = 21$
$x = 6$
Hence, 90% acid solution is 6 litres and 97% acid solution is 15 litres.

Solution 7:

Assume x kg of the first kind costing Rs. 250 per kg and y kg of the second kind costing Rs. 350 per kg sweets were bought.

It is estimated that 40 kg of sweets were needed.
$\Rightarrow x + y = 40$................(i)

The total budget for the sweets was Rs. 11,800.
$\Rightarrow 250x + 350y = 11,800$...............$(ii)$

Multiply equation $(i)$by 250, we get:

$$250x + 250y = 10000$$
$$250x + 350y = 11,800 \quad \text{[Equation } (ii)\text{]}$$
$$- \quad - \quad\quad - \quad \text{[Subtracting]}$$
$$-100y = -1800$$
$$\Rightarrow y = 18$$

Substituting $y = 18$ in equation (i), we get
$x + 18 = 40$
$\Rightarrow x = 22$

$\therefore$ 22 kgs of the first kind costing Rs. 250 per kg and 18 kgs of the second kind costing Rs. 350 per kg sweets were bought.

Solution 8:

Weight of Mr. Ahuja = x kg
and weight of Mrs. Ahuja = y kg.
After the dieting,
x - 5 = y
x - y = 5...(1)

and, $y - 4 = \frac{7}{8}x$

7x - 8y = -32...(2)
Multiplying equation no. (1) by 7, we get

$7x - 7y = 35 \quad \ldots(3)$

Now subtracting (2) from (3)

$$\begin{array}{l} 7x - 7y = 35 \quad \ldots(3) \\ 7x - 8y = -32 \quad \ldots(2) \\ - \quad + \quad\quad + \\ \hline \quad\quad y = 67 \end{array}$$

From (1)
x - 67 = 5 ⇒ x = 72
Thus, weight of Mr. Ahuja = 72 kg.
and that of Mr. Anuja = 67 kg.

Solution 9:

Let x be the constant expense per month of the family.
and y be the expense per month for a single member of the family.

For a family of 4 people,
the total monthly expense is Rs. 10,400.

$\Rightarrow x + 4y = 10,400 \ldots\ldots\ldots\ldots\ldots(i)$

For a family of 7 people,
the total monthly expense is Rs. 15,800.

$\Rightarrow x + 7y = 15,800 \ldots\ldots\ldots\ldots\ldots(ii)$

Subtracting equation (i) from equation (ii), we get:

$$\begin{array}{ll} x + 7y = 15800 & \\ x + 4y = 10400 & \text{[Equation } (i)\text{]} \\ - \quad - \quad\quad - & \text{[Subtracting]} \\ \hline \quad 3y = 5400 & \end{array}$$

$\Rightarrow y = 1800$

Substituting $y = 1800$ in equation (i), we get

$x + 4(1800) = 10,400$

$\Rightarrow x = 3200$

∴ The constant expense is Rs. 3,200 per month and
the monthly expense of each member of a family is Rs. 1,800

Solution 10:

Let the fixed charge be Rs. x and the charge per kilometer be Rs. y.

The charges for 10 km = Rs. 10y

The charges for 15 km = Rs. 15y

According to the question,

$x + 10y = 315$.....(i)

$x + 15y = 465$.....(ii)

Solving the equations, we get

$-5y = -150 \Rightarrow y = 30$

and $x = 315 - 10y = 315 - 10(30) = 15$

So, the fixed charges is Rs. 15 and the charges per kilometer is Rs. 30.

To travel 32 km, a person has to pay

Rs. 15 + Rs. 30(32) = Rs. 15 + Rs. 960 = Rs. 975

Solution 11:

Let the fixed charges be Rs. x and the charge for each extra day be Rs. y.

According to the question,

$x + 4y = 27$......(i)

and $x + 2y = 21$......(ii)

Solving the equations, we get

$2y = 6 \Rightarrow y = 3$

and $x = 21 - 2y = 21 - 2(3) = 15$

Hence, the fixed charges is Rs. 15 and the charge for each extra day is Rs. 3.

Solution 12:

Let the length of the rectangle be x units and the breadth of the rectangle be y units.
We know that, area of a rectangle = length × breadth = xy
According to the question,

$xy - 9 = (x - 5)(y + 3)$

$\Rightarrow xy - 9 = xy + 3x - 5y - 15$

$\Rightarrow 3x - 5y = 6 \ldots\ldots(i)$

$xy + 67 = (x + 3)(y + 2)$

$\Rightarrow xy + 67 = xy + 2x + 3y + 6$

$\Rightarrow 2x + 3y = 61 \ldots\ldots(ii)$

Multiply (i) by 2 and (ii) by 3, we get

$6x - 10y = 12 \ldots(iii)$

and $6x + 9y = 183 \ldots(iv)$

Solving (iii) and (iv), we get

$-19y = -171 \Rightarrow y = 9$

and $x = 17$

Hence, the length of the rectangle is 17 units and the breadth of the rectangle is 9 units.

Solution 13:

Let the pipe with larger diameter and smaller diameter be pipes A and B respectively.
Also, let pipe A work at a rate of x hours / unit and pipe B work at a rate of y hours / unit.
According to the question,

$x + y = \frac{1}{12} \Rightarrow 12x + 12y = 1 \ldots\ldots(i)$

and $4x + 9y = \frac{1}{2} \Rightarrow 8x + 18y = 1 \ldots\ldots(ii)$

Multiply (i) by 2 and (ii) by 3, we get

$24x + 24y = 2$ and $24x + 54y = 3$

On solving we get, $30y = 1 \Rightarrow y = \frac{1}{30}$

and $x = \frac{1}{20}$

Hence, the pipe with larger diameter will take 20 hours to fill the swimming pool and the pipe with smaller diameter will take 30 hours to fill the swimming pool.